PUZZLER

Soft contact lenses are comfortable to wear because they attract the proteins in the wearer's tears, incorporating the complex molecules right into the lenses. They become, in a sense, part of the wearer. Some types of makeup exploit this same attractive force to adhere to the skin. What is the nature of this force? *(Charles D. Winters)*

chapter

23 Electric Fields

Chapter Outline

23.1 Properties of Electric Charges
23.2 Insulators and Conductors
23.3 Coulomb's Law
23.4 The Electric Field
23.5 Electric Field of a Continuous Charge Distribution
23.6 Electric Field Lines
23.7 Motion of Charged Particles in a Uniform Electric Field

The electromagnetic force between charged particles is one of the fundamental forces of nature. We begin this chapter by describing some of the basic properties of electric forces. We then discuss Coulomb's law, which is the fundamental law governing the force between any two charged particles. Next, we introduce the concept of an electric field associated with a charge distribution and describe its effect on other charged particles. We then show how to use Coulomb's law to calculate the electric field for a given charge distribution. We conclude the chapter with a discussion of the motion of a charged particle in a uniform electric field.

23.1 PROPERTIES OF ELECTRIC CHARGES

11.2 A number of simple experiments demonstrate the existence of electric forces and charges. For example, after running a comb through your hair on a dry day, you will find that the comb attracts bits of paper. The attractive force is often strong enough to suspend the paper. The same effect occurs when materials such as glass or rubber are rubbed with silk or fur.

Another simple experiment is to rub an inflated balloon with wool. The balloon then adheres to a wall, often for hours. When materials behave in this way, they are said to be *electrified,* or to have become **electrically charged.** You can easily electrify your body by vigorously rubbing your shoes on a wool rug. The electric charge on your body can be felt and removed by lightly touching (and startling) a friend. Under the right conditions, you will see a spark when you touch, and both of you will feel a slight tingle. (Experiments such as these work best on a dry day because an excessive amount of moisture in the air can cause any charge you build up to "leak" from your body to the Earth.)

In a series of simple experiments, it is found that there are two kinds of electric charges, which were given the names **positive** and **negative** by Benjamin Franklin (1706–1790). To verify that this is true, consider a hard rubber rod that has been rubbed with fur and then suspended by a nonmetallic thread, as shown in Figure 23.1. When a glass rod that has been rubbed with silk is brought near the rubber rod, the two attract each other (Fig. 23.1a). On the other hand, if two charged rubber rods (or two charged glass rods) are brought near each other, as shown in Figure 23.1b, the two repel each other. This observation shows that the rubber and glass are in two different states of electrification. On the basis of these observations, we conclude that **like charges repel one another and unlike charges attract one another.**

QuickLab

Rub an inflated balloon against your hair and then hold the balloon near a thin stream of water running from a faucet. What happens? (A rubbed plastic pen or comb will also work.)

Using the convention suggested by Franklin, the electric charge on the glass rod is called positive and that on the rubber rod is called negative. Therefore, any charged object attracted to a charged rubber rod (or repelled by a charged glass rod) must have a positive charge, and any charged object repelled by a charged rubber rod (or attracted to a charged glass rod) must have a negative charge.

Attractive electric forces are responsible for the behavior of a wide variety of commercial products. For example, the plastic in many contact lenses, *etafilcon,* is made up of molecules that electrically attract the protein molecules in human tears. These protein molecules are absorbed and held by the plastic so that the lens ends up being primarily composed of the wearer's tears. Because of this, the wearer's eye does not treat the lens as a foreign object, and it can be worn comfortably. Many cosmetics also take advantage of electric forces by incorporating materials that are electrically attracted to skin or hair, causing the pigments or other chemicals to stay put once they are applied.

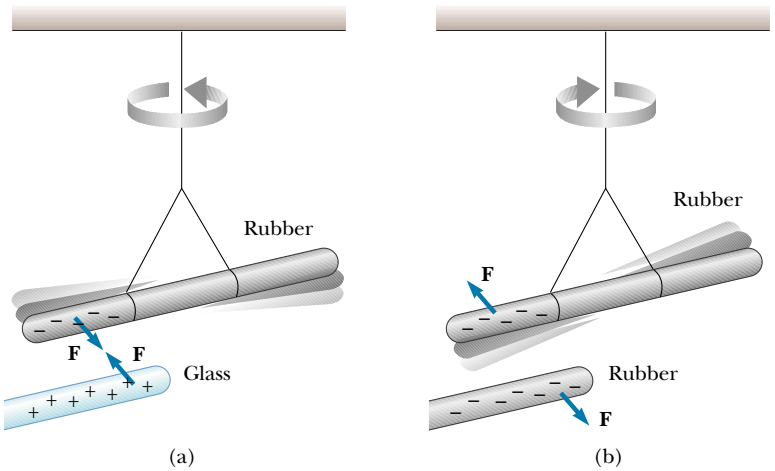


Figure 23.1 (a) A negatively charged rubber rod suspended by a thread is attracted to a positively charged glass rod. (b) A negatively charged rubber rod is repelled by another negatively charged rubber rod.

Charge is conserved

Another important aspect of Franklin's model of electricity is the implication that **electric charge is always conserved.** That is, when one object is rubbed against another, charge is not created in the process. The electrified state is due to a *transfer* of charge from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge. For example, when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod. We now know from our understanding of atomic structure that negatively charged electrons are transferred from the glass to the silk in the rubbing process. Similarly, when rubber is rubbed with fur, electrons are transferred from the fur to the rubber, giving the rubber a net negative charge and the fur a net positive charge. This process is consistent with the fact that neutral, uncharged matter contains as many positive charges (protons within atomic nuclei) as negative charges (electrons).

Figure 23.2 Rubbing a balloon against your hair on a dry day causes the balloon and your hair to become charged.

Quick Quiz 23.1

If you rub an inflated balloon against your hair, the two materials attract each other, as shown in Figure 23.2. Is the amount of charge present in the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

Charge is quantized

In 1909, Robert Millikan (1868–1953) discovered that electric charge always occurs as some integral multiple of a fundamental amount of charge e. In modern terms, the electric charge q is said to be **quantized,** where q is the standard symbol used for charge. That is, electric charge exists as discrete "packets," and we can write $q = Ne$, where N is some integer. Other experiments in the same period showed that the electron has a charge $-e$ and the proton has a charge of equal magnitude but opposite sign $+e$. Some particles, such as the neutron, have no charge. A neutral atom must contain as many protons as electrons.

Because charge is a conserved quantity, the net charge in a closed region remains the same. If charged particles are created in some process, they are always created in pairs whose members have equal-magnitude charges of opposite sign.

From our discussion thus far, we conclude that electric charge has the following important properties:

Properties of electric charge

- Two kinds of charges occur in nature, with the property that unlike charges attract one another and like charges repel one another.
- Charge is conserved.
- Charge is quantized.

23.2 INSULATORS AND CONDUCTORS

11.3 It is convenient to classify substances in terms of their ability to conduct electric charge:

Electrical **conductors** are materials in which electric charges move freely, whereas electrical **insulators** are materials in which electric charges cannot move freely.

Materials such as glass, rubber, and wood fall into the category of electrical insulators. When such materials are charged by rubbing, only the area rubbed becomes charged, and the charge is unable to move to other regions of the material.

Metals are good conductors

In contrast, materials such as copper, aluminum, and silver are good electrical conductors. When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material. If you hold a copper rod in your hand and rub it with wool or fur, it will not attract a small piece of paper. This might suggest that a metal cannot be charged. However, if you attach a wooden handle to the rod and then hold it by that handle as you rub the rod, the rod will remain charged and attract the piece of paper. The explanation for this is as follows: Without the insulating wood, the electric charges produced by rubbing readily move from the copper through your body and into the Earth. The insulating wooden handle prevents the flow of charge into your hand.

Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors. Silicon and germanium are well-known examples of semiconductors commonly used in the fabrication of a variety of electronic devices, such as transistors and light-emitting diodes. The electrical properties of semiconductors can be changed over many orders of magnitude by the addition of controlled amounts of certain atoms to the materials.

When a conductor is connected to the Earth by means of a conducting wire or pipe, it is said to be **grounded.** The Earth can then be considered an infinite "sink" to which electric charges can easily migrate. With this in mind, we can understand how to charge a conductor by a process known as **induction.**

To understand induction, consider a neutral (uncharged) conducting sphere insulated from ground, as shown in Figure 23.3a. When a negatively charged rubber rod is brought near the sphere, the region of the sphere nearest the rod obtains an excess of positive charge while the region farthest from the rod obtains an equal excess of negative charge, as shown in Figure 23.3b. (That is, electrons in the region nearest the rod migrate to the opposite side of the sphere. This occurs even if the rod never actually touches the sphere.) If the same experiment is performed with a conducting wire connected from the sphere to ground (Fig. 23.3c), some of the electrons in the conductor are so strongly repelled by the presence of

Charging by induction

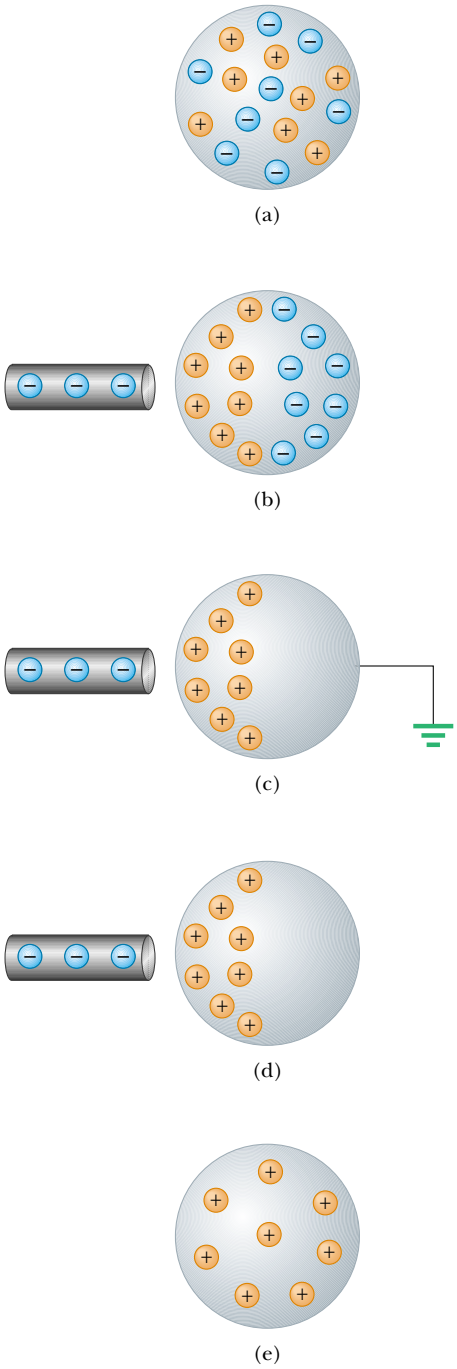
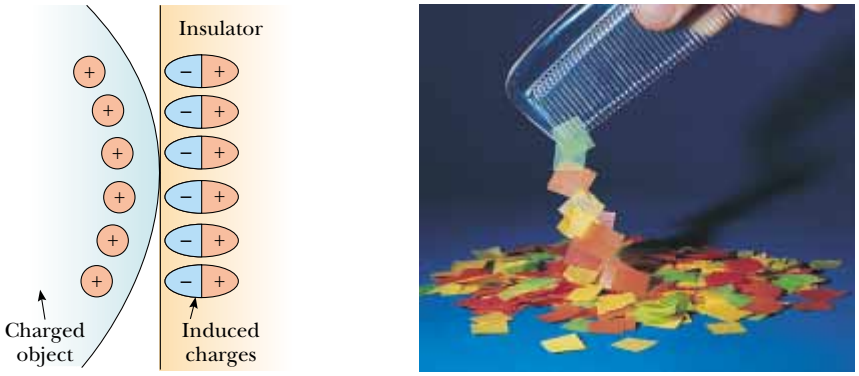


Figure 23.3 Charging a metallic object by *induction* (that is, the two objects never touch each other). (a) A neutral metallic sphere, with equal numbers of positive and negative charges. (b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed near the sphere. (c) When the sphere is grounded, some of its electrons leave through the ground wire. (d) When the ground connection is removed, the sphere has excess positive charge that is nonuniformly distributed. (e) When the rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.



(a) (b)

Figure 23.4 (a) The charged object on the left induces charges on the surface of an insulator. (b) A charged comb attracts bits of paper because charges are displaced in the paper.

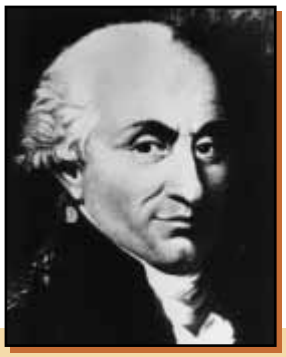
QuickLab

Tear some paper into very small pieces. Comb your hair and then bring the comb close to the paper pieces. Notice that they are accelerated toward the comb. How does the magnitude of the electric force compare with the magnitude of the gravitational force exerted on the paper? Keep watching and you might see a few pieces jump away from the comb. They don't just fall away; they are repelled. What causes this?

the negative charge in the rod that they move out of the sphere through the ground wire and into the Earth. If the wire to ground is then removed (Fig. 23.3d), the conducting sphere contains an excess of *induced* positive charge. When the rubber rod is removed from the vicinity of the sphere (Fig. 23.3e), this induced positive charge remains on the ungrounded sphere. Note that the charge remaining on the sphere is uniformly distributed over its surface because of the repulsive forces among the like charges. Also note that the rubber rod loses none of its negative charge during this process.

Charging an object by induction requires no contact with the body inducing the charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.

A process similar to induction in conductors takes place in insulators. In most neutral molecules, the center of positive charge coincides with the center of negative charge. However, in the presence of a charged object, these centers inside each molecule in an insulator may shift slightly, resulting in more positive charge on one side of the molecule than on the other. This realignment of charge within individual molecules produces an induced charge on the surface of the insulator, as shown in Figure 23.4. Knowing about induction in insulators, you should be able to explain why a comb that has been rubbed through hair attracts bits of electrically neutral paper and why a balloon that has been rubbed against your clothing is able to stick to an electrically neutral wall.



Charles Coulomb (1736–1806) Coulomb's major contribution to science was in the field of electrostatics and magnetism. During his lifetime, he also investigated the strengths of materials and determined the forces that affect objects on beams, thereby contributing to the field of structural mechanics. In the field of ergonomics, his research provided a fundamental understanding of the ways in which people and animals can best do work. *(Photo courtesy of AIP Niels Bohr Library/E. Scott Barr Collection)*

Quick Quiz 23.2

Object A is attracted to object B. If object B is known to be positively charged, what can we say about object A? (a) It is positively charged. (b) It is negatively charged. (c) It is electrically neutral. (d) Not enough information to answer.

23.3 COULOMB'S LAW

11.4 Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance, which he invented (Fig. 23.5).

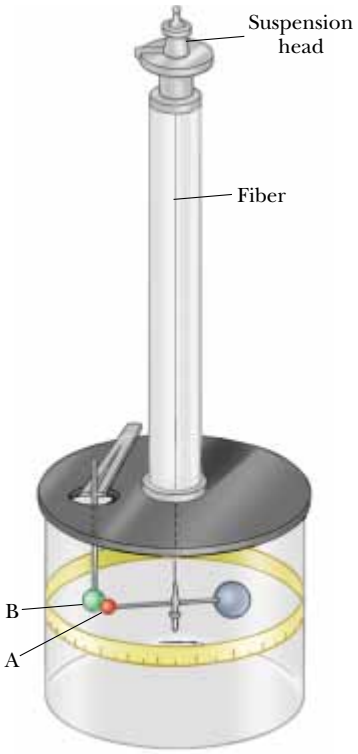


Figure 23.5 Coulomb's torsion balance, used to establish the inverse-square law for the electric force between two charges.

Coulomb confirmed that the electric force between two small charged spheres is proportional to the inverse square of their separation distance r—that is, $F_e \propto 1/r^2$. The operating principle of the torsion balance is the same as that of the apparatus used by Cavendish to measure the gravitational constant (see Section 14.2), with the electrically neutral spheres replaced by charged ones. The electric force between charged spheres A and B in Figure 23.5 causes the spheres to either attract or repel each other, and the resulting motion causes the suspended fiber to twist. Because the restoring torque of the twisted fiber is proportional to the angle through which the fiber rotates, a measurement of this angle provides a quantitative measure of the electric force of attraction or repulsion. Once the spheres are charged by rubbing, the electric force between them is very large compared with the gravitational attraction, and so the gravitational force can be neglected.

Coulomb's experiments showed that the **electric force** between two stationary charged particles

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, we can express **Coulomb's law** as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

$$F_e = k_e \frac{|q_1||q_2|}{r^2} \tag{23.1}$$

where k_e is a constant called the **Coulomb constant.** In his experiments, Coulomb was able to show that the value of the exponent of r was 2 to within an uncertainty of a few percent. Modern experiments have shown that the exponent is 2 to within an uncertainty of a few parts in 10^{16}.

The value of the Coulomb constant depends on the choice of units. The SI unit of charge is the **coulomb** (C). The Coulomb constant k_e in SI units has the value

Coulomb constant

$$k_e = 8.987\,5 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 (lowercase Greek epsilon) is known as the *permittivity of free space* and has the value $8.854\,2 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$.

The smallest unit of charge known in nature is the charge on an electron or proton,[1] which has an absolute value of

Charge on an electron or proton

$$|e| = 1.602\,19 \times 10^{-19}\ \mathrm{C}$$

Therefore, 1 C of charge is approximately equal to the charge of 6.24×10^{18} electrons or protons. This number is very small when compared with the number of

[1] No unit of charge smaller than e has been detected as a free charge; however, recent theories propose the existence of particles called *quarks* having charges $e/3$ and $2e/3$. Although there is considerable experimental evidence for such particles inside nuclear matter, *free* quarks have never been detected. We discuss other properties of quarks in Chapter 46 of the extended version of this text.

TABLE 23.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

free electrons[2] in 1 cm^3 of copper, which is of the order of 10^{23}. Still, 1 C is a substantial amount of charge. In typical experiments in which a rubber or glass rod is charged by friction, a net charge of the order of 10^{-6} C is obtained. In other words, only a very small fraction of the total available charge is transferred between the rod and the rubbing material.

The charges and masses of the electron, proton, and neutron are given in Table 23.1.

EXAMPLE 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e \frac{|e|^2}{r^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\ \text{N}}$$

Using Newton's law of gravitation and Table 23.1 for the particle masses, we find that the gravitational force has the magnitude

$$F_g = G \frac{m_e m_p}{r^2}$$

$$= \left(6.7 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= \boxed{3.6 \times 10^{-47}\ \text{N}}$$

The ratio $F_e/F_g \approx 2 \times 10^{39}$. Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces. Other than magnitude, what is a fundamental difference between the two forces?

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. Thus, the law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \qquad (23.2)$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2, as shown in Figure 23.6a. Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is

[2] A metal atom, such as copper, contains one or more outer electrons, which are weakly bound to the nucleus. When many atoms combine to form a metal, the so-called *free electrons* are these outer electrons, which are not bound to any one atom. These electrons move about the metal in a manner similar to that of gas molecules moving in a container.

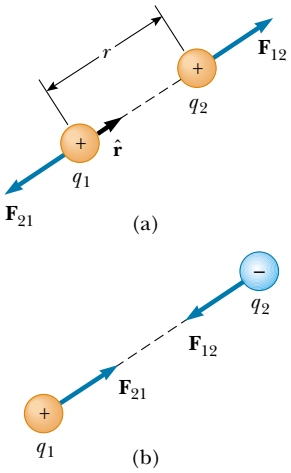


Figure 23.6 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\mathbf{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\mathbf{F}_{12}$ exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is, $\mathbf{F}_{21} = -\mathbf{F}_{12}$. Finally, from Equation 23.2, we see that if q_1 and q_2 have the same sign, as in Figure 23.6a, the product $q_1 q_2$ is positive and the force is repulsive. If q_1 and q_2 are of opposite sign, as shown in Figure 23.6b, the product $q_1 q_2$ is negative and the force is attractive. Noting the sign of the product $q_1 q_2$ is an easy way of determining the direction of forces acting on the charges.

Quick Quiz 23.3

Object A has a charge of $+2\ \mu$C, and object B has a charge of $+6\ \mu$C. Which statement is true?

(a) $\mathbf{F}_{AB} = -3\mathbf{F}_{BA}$. (b) $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$. (c) $3\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

When more than two charges are present, the force between any pair of them is given by Equation 23.2. Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

Example 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.7, where $q_1 = q_3 = 5.0\ \mu$C, $q_2 = -2.0\ \mu$C, and $a = 0.10$ m. Find the resultant force exerted on q_3.

Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force F_{23} exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force F_{13} exerted by q_1 on q_3 is repulsive because both charges are positive.

The magnitude of $\mathbf{F}_{23}$ is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(2.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(0.10\ \text{m})^2}$$

$$= 9.0\ \text{N}$$

Note that because q_3 and q_2 have opposite signs, $\mathbf{F}_{23}$ is to the left, as shown in Figure 23.7.

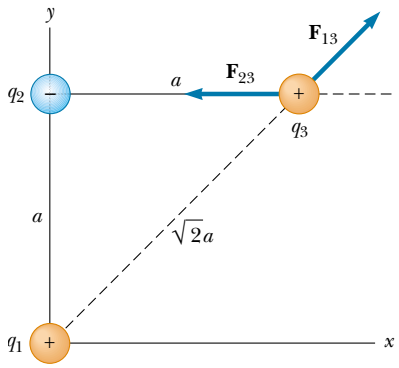


Figure 23.7 The force exerted by q_1 on q_3 is $\mathbf{F}_{13}$. The force exerted by q_2 on q_3 is $\mathbf{F}_{23}$. The resultant force $\mathbf{F}_3$ exerted on q_3 is the vector sum $\mathbf{F}_{13} + \mathbf{F}_{23}$.

The magnitude of the force exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{2(0.10\ \text{m})^2}$$

$$= 11\ \text{N}$$

The force $\mathbf{F}_{13}$ is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of $\mathbf{F}_{13}$ are equal, with magnitude given by $F_{13} \cos 45° = 7.9$ N.

The force $\mathbf{F}_{23}$ is in the negative x direction. Hence, the x and y components of the resultant force acting on q_3 are

$$F_{3x} = F_{13x} + F_{23} = 7.9\ \text{N} - 9.0\ \text{N} = -1.1\ \text{N}$$

$$F_{3y} = F_{13y} = 7.9\ \text{N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

$$\mathbf{F}_3 = (-1.1\mathbf{i} + 7.9\mathbf{j})\ \text{N}$$

Exercise Find the magnitude and direction of the resultant force $\mathbf{F}_3$.

Answer 8.0 N at an angle of 98° with the x axis.

EXAMPLE 23.3 Where Is the Resultant Force Zero?

Three point charges lie along the x axis as shown in Figure 23.8. The positive charge $q_1 = 15.0\ \mu\text{C}$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\ \mu\text{C}$ is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3?

Solution Because q_3 is negative and q_1 and q_2 are positive, the forces $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are both attractive, as indicated in Figure 23.8. From Coulomb's law, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ have magnitudes

$$F_{13} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2} \qquad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

For the resultant force on q_3 to be zero, $\mathbf{F}_{23}$ must be equal in magnitude and opposite in direction to $\mathbf{F}_{13}$, or

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2.00 - x)^2}$$

Noting that k_e and q_3 are common to both sides and so can be dropped, we solve for x and find that

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \text{C}) = x^2(15.0 \times 10^{-6}\ \text{C})$$

Solving this quadratic equation for x, we find that $x = 0.775$ m. Why is the negative root not acceptable?

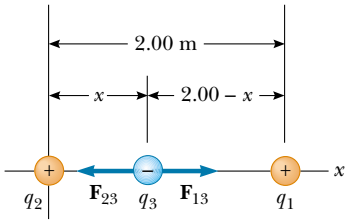


Figure 23.8 Three point charges are placed along the x axis. If the net force acting on q_3 is zero, then the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force $\mathbf{F}_{23}$ exerted by q_2 on q_3.

EXAMPLE 23.4 Find the Charge on the Spheres

Two identical small charged spheres, each having a mass of 3.0×10^{-2} kg, hang in equilibrium as shown in Figure 23.9a. The length of each string is 0.15 m, and the angle θ is 5.0°. Find the magnitude of the charge on each sphere.

Solution From the right triangle shown in Figure 23.9a, we see that $\sin\theta = a/L$. Therefore,

$$a = L \sin\theta = (0.15\ \text{m}) \sin 5.0° = 0.013\ \text{m}$$

The separation of the spheres is $2a = 0.026$ m.

The forces acting on the left sphere are shown in Figure 23.9b. Because the sphere is in equilibrium, the forces in the

horizontal and vertical directions must separately add up to zero:

$$(1)\qquad \sum F_x = T\sin\theta - F_e = 0$$

$$(2)\qquad \sum F_y = T\cos\theta - mg = 0$$

From Equation (2), we see that $T = mg/\cos\theta$; thus, T can be eliminated from Equation (1) if we make this substitution. This gives a value for the magnitude of the electric force F_e:

$$(3)\qquad F_e = mg\tan\theta$$
$$= (3.0\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)\tan 5.0^\circ$$
$$= 2.6\times10^{-2}\ \text{N}$$

From Coulomb's law (Eq. 23.1), the magnitude of the electric force is

$$F_e = k_e\frac{|q|^2}{r^2}$$

where $r = 2a = 0.026$ m and $|q|$ is the magnitude of the charge on each sphere. (Note that the term $|q|^2$ arises here because the charge is the same on both spheres.) This equation can be solved for $|q|^2$ to give

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6\times10^{-2}\ \text{N})(0.026\ \text{m})^2}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}$$

$$|q| = 4.4\times10^{-8}\ \text{C}$$

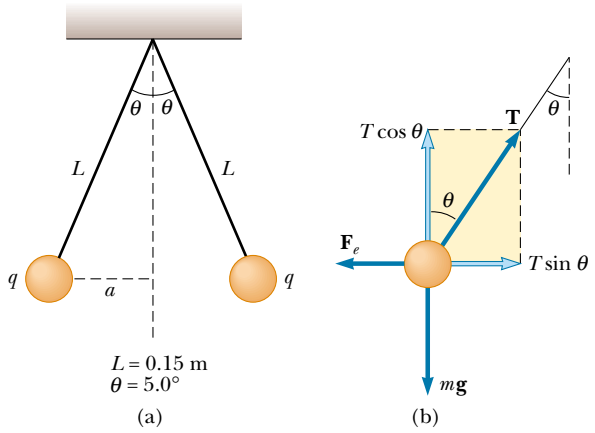


Figure 23.9 (a) Two identical spheres, each carrying the same charge q, suspended in equilibrium. (b) The free-body diagram for the sphere on the left.

Exercise If the charge on the spheres were negative, how many electrons would have to be added to them to yield a net charge of -4.4×10^{-8} C?

Answer 2.7×10^{11} electrons.

QuickLab

For this experiment you need two 20-cm strips of transparent tape (mass of each ≈ 65 mg). Fold about 1 cm of tape over at one end of each strip to create a handle. Press both pieces of tape side by side onto a table top, rubbing your finger back and forth across the strips. Quickly pull the strips off the surface so that they become charged. Hold the tape handles together and the strips will repel each other, forming an inverted "V" shape. Measure the angle between the pieces, and estimate the excess charge on each strip. Assume that the charges act as if they were located at the center of mass of each strip.

23.4 THE ELECTRIC FIELD

11.5 Two field forces have been introduced into our discussions so far—the gravitational force and the electric force. As pointed out earlier, field forces can act through space, producing an effect even when no physical contact between the objects occurs. The gravitational field **g** at a point in space was defined in Section 14.6 to be equal to the gravitational force $\mathbf{F}_g$ acting on a test particle of mass m divided by that mass: $\mathbf{g} \equiv \mathbf{F}_g/m$. A similar approach to electric forces was developed by Michael Faraday and is of such practical value that we shall devote much attention to it in the next several chapters. In this approach, an **electric field** is said to exist in the region of space around a charged object. When another charged object enters this electric field, an electric force acts on it. As an example, consider Figure 23.10, which shows a small positive test charge q_0 placed near a second object carrying a much greater positive charge Q. We define the strength (in other words, the magnitude) of the electric field at the location of the test charge to be the electric force *per unit charge,* or to be more specific

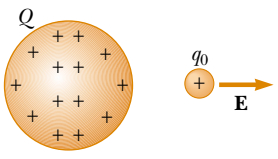


Figure 23.10 A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field **E** directed as shown.

the electric field E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \tag{23.3}$$

Definition of electric field

Note that **E** is the field produced by some charge *external* to the test charge—it is not the field produced by the test charge itself. Also, note that the existence of an electric field is a property of its source. For example, every electron comes with its own electric field.

The vector **E** has the SI units of newtons per coulomb (N/C), and, as Figure 23.10 shows, its direction is the direction of the force a positive test charge experiences when placed in the field. We say that **an electric field exists at a point if a test charge at rest at that point experiences an electric force.** Once the magnitude and direction of the electric field are known at some point, the electric force exerted on *any* charged particle placed at that point can be calculated from

This dramatic photograph captures a lightning bolt striking a tree near some rural homes.

TABLE 23.2 Typical Electric Field Values

Source	E (N/C)
Fluorescent lighting tube	10
Atmosphere (fair weather)	100
Balloon rubbed on hair	1 000
Atmosphere (under thundercloud)	10 000
Photocopier	100 000
Spark in air	$>3\,000\,000$
Near electron in hydrogen atom	5×10^{11}

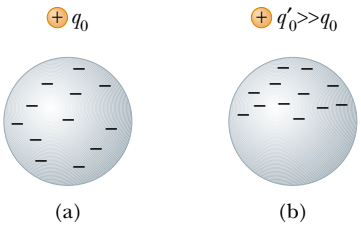


Figure 23.11 (a) For a small enough test charge q_0, the charge distribution on the sphere is undisturbed. (b) When the test charge q_0' is greater, the charge distribution on the sphere is disturbed as the result of the proximity of q_0'.

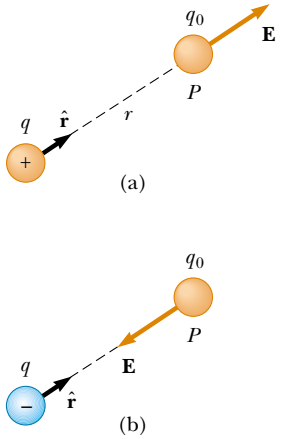


Figure 23.12 A test charge q_0 at point P is a distance r from a point charge q. (a) If q is positive, then the electric field at P points radially outward from q. (b) If q is negative, then the electric field at P points radially inward toward q.

Equation 23.3. Furthermore, the electric field is said to exist at some point (even empty space) **regardless of whether a test charge is located at that point.** (This is analogous to the gravitational field set up by any object, which is said to exist at a given point regardless of whether some other object is present at that point to "feel" the field.) The electric field magnitudes for various field sources are given in Table 23.2.

When using Equation 23.3, we must assume that the test charge q_0 is small enough that it does not disturb the charge distribution responsible for the electric field. If a vanishingly small test charge q_0 is placed near a uniformly charged metallic sphere, as shown in Figure 23.11a, the charge on the metallic sphere, which produces the electric field, remains uniformly distributed. If the test charge is great enough ($q_0' \gg q_0$), as shown in Figure 23.11b, the charge on the metallic sphere is redistributed and the ratio of the force to the test charge is different: $(F_e'/q_0' \neq F_e/q_0)$. That is, because of this redistribution of charge on the metallic sphere, the electric field it sets up is different from the field it sets up in the presence of the much smaller q_0.

To determine the direction of an electric field, consider a point charge q located a distance r from a test charge q_0 located at a point P, as shown in Figure 23.12. According to Coulomb's law, the force exerted by q on the test charge is

$$\mathbf{F}_e = k_e \frac{qq_0}{r^2}\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q toward q_0. Because the electric field at P, the position of the test charge, is defined by $\mathbf{E} = \mathbf{F}_e/q_0$, we find that at P, the electric field created by q is

$$\mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}} \qquad (23.4)$$

If q is positive, as it is in Figure 23.12a, the electric field is directed radially outward from it. If q is negative, as it is in Figure 23.12b, the field is directed toward it.

To calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually using Equation 23.4 and then add them vectorially. In other words,

> at any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges.

This superposition principle applied to fields follows directly from the superposition property of electric forces. Thus, the electric field of a group of charges can

This metallic sphere is charged by a generator so that it carries a net electric charge. The high concentration of charge on the sphere creates a strong electric field around the sphere. The charges then leak through the gas surrounding the sphere, producing a pink glow.

be expressed as

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \qquad (23.5)$$

where r_i is the distance from the ith charge q_i to the point P (the location of the test charge) and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Quick Quiz 23.4

A charge of $+3\ \mu$C is at a point P where the electric field is directed to the right and has a magnitude of 4×10^6 N/C. If the charge is replaced with a -3-μC charge, what happens to the electric field at P?

Example 23.5 Electric Field Due to Two Charges

A charge $q_1 = 7.0\ \mu$C is located at the origin, and a second charge $q_2 = -5.0\ \mu$C is located on the x axis, 0.30 m from the origin (Fig. 23.13). Find the electric field at the point P, which has coordinates (0, 0.40) m.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.13. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(7.0 \times 10^{-6}\ \text{C})}{(0.40\ \text{m})^2}$$
$$= 3.9 \times 10^5\ \text{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(5.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$
$$= 1.8 \times 10^5\ \text{N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

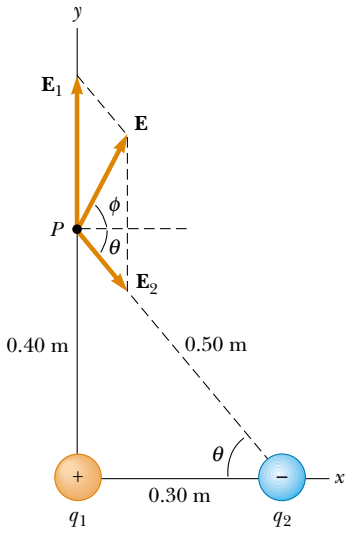


Figure 23.13 The total electric field $\mathbf{E}$ at P equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ is the field due to the positive charge q_1 and $\mathbf{E}_2$ is the field due to the negative charge q_2.

$$\mathbf{E}_1 = 3.9 \times 10^5 \mathbf{j}\ \text{N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} - 1.4 \times 10^5 \mathbf{j})\ \text{N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \mathbf{i} + 2.5 \times 10^5 \mathbf{j})\ \text{N/C}$$

From this result, we find that $\mathbf{E}$ has a magnitude of 2.7×10^5 N/C and makes an angle ϕ of 66° with the positive x axis.

Exercise Find the electric force exerted on a charge of 2.0×10^{-8} C located at P.

Answer 5.4×10^{-3} N in the same direction as $\mathbf{E}$.

EXAMPLE 23.6 Electric Field of a Dipole

An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by some distance. For the dipole shown in Figure 23.14, find the electric field $\mathbf{E}$ at P due to the charges, where P is a distance $y \gg a$ from the origin.

Solution At P, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude because P is equidistant from the charges. The total field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

The y components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other, and the x components add because they are both in the positive x direction. Therefore, $\mathbf{E}$ is parallel to the x axis and has a magnitude equal to $2E_1 \cos\theta$. From Figure 23.14 we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$. Therefore,

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$

Because $y \gg a$, we can neglect a^2 and write

$$E \approx k_e \frac{2qa}{y^3}$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as $1/r^3$, whereas the more slowly varying field of a point charge varies as $1/r^2$ (see Eq. 23.4). This is because at distant points, the fields of the two charges of equal magnitude and opposite sign almost cancel each other. The $1/r^3$ variation in E for the dipole also is obtained for a distant point along the x axis (see Problem 21) and for any general distant point.

The electric dipole is a good model of many molecules, such as hydrochloric acid (HCl). As we shall see in later chapters, neutral atoms and molecules behave as dipoles when placed in an external electric field. Furthermore, many molecules, such as HCl, are permanent dipoles. The effect of such dipoles on the behavior of materials subjected to electric fields is discussed in Chapter 26.

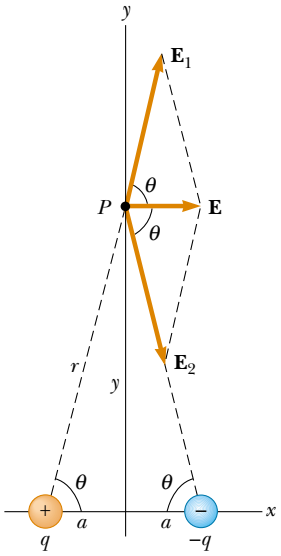


Figure 23.14 The total electric field $\mathbf{E}$ at P due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$. The field $\mathbf{E}_1$ is due to the positive charge q, and $\mathbf{E}_2$ is the field due to the negative charge $-q$.

23.5 ELECTRIC FIELD OF A CONTINUOUS CHARGE DISTRIBUTION

Very often the distances between charges in a group of charges are much smaller than the distance from the group to some point of interest (for example, a point where the electric field is to be calculated). In such situations, the system of

charges is smeared out, or *continuous*. That is, the system of closely spaced charges is equivalent to a total charge that is continuously distributed along some line, over some surface, or throughout some volume.

To evaluate the electric field created by a continuous charge distribution, we use the following procedure: First, we divide the charge distribution into small elements, each of which contains a small charge Δq, as shown in Figure 23.15. Next, we use Equation 23.4 to calculate the electric field due to one of these elements at a point P. Finally, we evaluate the total field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

A continuous charge distribution

The electric field at P due to one element carrying charge Δq is

$$\Delta \mathbf{E} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

where r is the distance from the element to point P and $\hat{\mathbf{r}}$ is a unit vector directed from the charge element toward P. The total electric field at P due to all elements in the charge distribution is approximately

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where the index i refers to the ith element in the distribution. Because the charge distribution is approximately continuous, the total field at P in the limit $\Delta q_i \to 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \qquad \textbf{(23.6)}$$

Electric field of a continuous charge distribution

where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately.

We illustrate this type of calculation with several examples, in which we assume the charge is uniformly distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a charge density along with the following notations:

- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

$$\rho \equiv \frac{Q}{V}$$

Volume charge density

where ρ has units of coulombs per cubic meter ($\mathrm{C/m^3}$).

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (lowercase Greek sigma) is defined by

$$\sigma \equiv \frac{Q}{A}$$

Surface charge density

where σ has units of coulombs per square meter ($\mathrm{C/m^2}$).

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

Linear charge density

where λ has units of coulombs per meter (C/m).

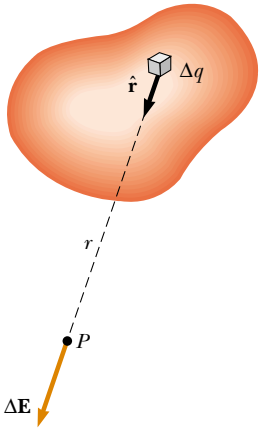


Figure 23.15 The electric field at P due to a continuous charge distribution is the vector sum of the fields $\Delta \mathbf{E}$ due to all the elements Δq of the charge distribution.

- If the charge is nonuniformly distributed over a volume, surface, or line, we have to express the charge densities as

$$\rho = \frac{dQ}{dV} \qquad \sigma = \frac{dQ}{dA} \qquad \lambda = \frac{dQ}{d\ell}$$

where dQ is the amount of charge in a small volume, surface, or length element.

EXAMPLE 23.7 The Electric Field Due to a Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig. 23.16).

Solution Let us assume that the rod is lying along the x axis, that dx is the length of one small segment, and that dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$.

The field $d\mathbf{E}$ due to this segment at P is in the negative x direction (because the source of the field carries a positive charge Q), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e \lambda \frac{dx}{x^2}$$

Because every other element also produces a field in the negative x direction, the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P, is given by Equation 23.6, which in this case becomes[3]

$$E = \int_a^{\ell+a} k_e \lambda \frac{dx}{x^2}$$

where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x = \ell + a$). The constants k_e and λ can be removed from the integral to yield

$$E = k_e \lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell+a}$$

$$= k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \frac{k_e Q}{a(\ell + a)}$$

where we have used the fact that the total charge $Q = \lambda \ell$.

If P is far from the rod ($a \gg \ell$), then the ℓ in the denominator can be neglected, and $E \approx k_e Q / a^2$. This is just the form you would expect for a point charge. Therefore, at large values of a/ℓ, the charge distribution appears to be a point charge of magnitude Q. The use of the limiting technique ($a/\ell \rightarrow \infty$) often is a good method for checking a theoretical formula.

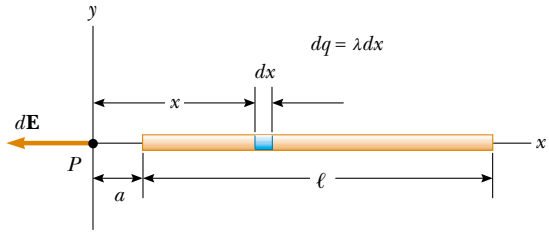


Figure 23.16 The electric field at P due to a uniformly charged rod lying along the x axis. The magnitude of the field at P due to the segment of charge dq is $k_e dq/x^2$. The total field at P is the vector sum over all segments of the rod.

EXAMPLE 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.17a).

Solution The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the axis and a component $dE_\perp$ perpendicular to the axis. As we see in Figure 23.17b, however, the resultant field at P must lie along the x axis because the perpendicular components of all the

[3] It is important that you understand how to carry out integrations such as this. First, express the charge element dq in terms of the other variables in the integral (in this example, there is one variable, x, and so we made the change $dq = \lambda\, dx$). The integral must be over scalar quantities; therefore, you must express the electric field in terms of components, if necessary. (In this example the field has only an x component, so we do not bother with this detail.) Then, reduce your expression to an integral over a single variable (or to multiple integrals, each over a single variable). In examples that have spherical or cylindrical symmetry, the single variable will be a radial coordinate.

various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that

$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\int dq$$

$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\,Q$$

This result shows that the field is zero at $x = 0$. Does this finding surprise you?

Exercise Show that at great distances from the ring ($x \gg a$) the electric field along the axis shown in Figure 23.17 approaches that of a point charge of magnitude Q.

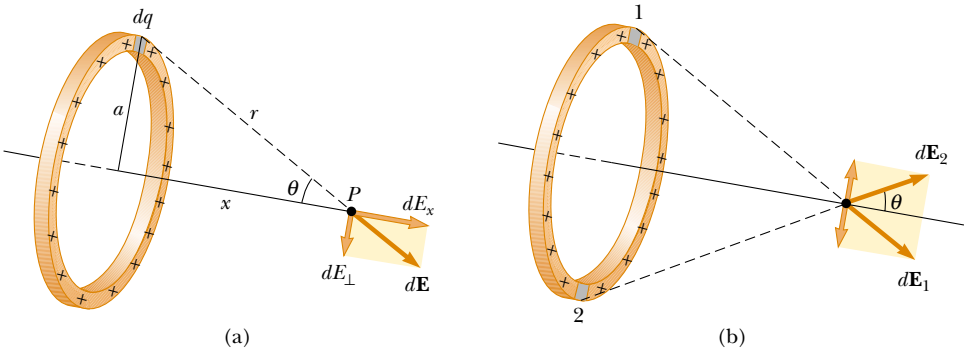


Figure 23.17 A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. The perpendicular component of the field at P due to segment 1 is canceled by the perpendicular component due to segment 2.

EXAMPLE 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk (Fig. 23.18).

Solution If we consider the disk as a set of concentric rings, we can use our result from Example 23.8—which gives the field created by a ring of radius a—and sum the contributions of all rings making up the disk. By symmetry, the field at an axial point must be along the central axis.

The ring of radius r and width dr shown in Figure 23.18 has a surface area equal to $2\pi r\,dr$. The charge dq on this ring is equal to the area of the ring multiplied by the surface charge density: $dq = 2\pi\sigma r\,dr$. Using this result in the equation given for E_x in Example 23.8 (with a replaced by r), we have for the field due to the ring

$$dE = \frac{k_e x}{(x^2 + r^2)^{3/2}}\,(2\pi\sigma r\,dr)$$

To obtain the total field at P, we integrate this expression over the limits $r = 0$ to $r = R$, noting that x is a constant. This gives

$$E = k_e x\pi\sigma\int_0^R \frac{2r\,dr}{(x^2 + r^2)^{3/2}}$$

$$= k_e x\pi\sigma\int_0^R (x^2 + r^2)^{-3/2}\,d(r^2)$$

$$= k_e x\pi\sigma\left[\frac{(x^2 + r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$= 2\pi k_e\sigma\left(\frac{x}{|x|} - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

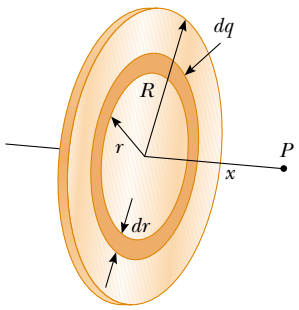


Figure 23.18 A uniformly charged disk of radius R. The electric field at an axial point P is directed along the central axis, perpendicular to the plane of the disk.

This result is valid for all values of x. We can calculate the field close to the disk along the axis by assuming that $R \gg x$; thus, the expression in parentheses reduces to unity:

$$E \approx 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

where $\epsilon_0 = 1/(4\pi k_e)$ is the permittivity of free space. As we shall find in the next chapter, we obtain the same result for the field created by a uniformly charged infinite sheet.

23.6 ELECTRIC FIELD LINES

11.5

A convenient way of visualizing electric field patterns is to draw lines that follow the same direction as the electric field vector at any point. These lines, called **electric field lines,** are related to the electric field in any region of space in the following manner:

- The electric field vector **E** is tangent to the electric field line at each point.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, E is great when the field lines are close together and small when they are far apart.

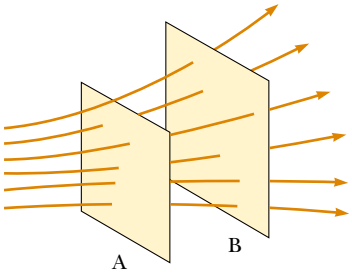


Figure 23.19 Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

These properties are illustrated in Figure 23.19. The density of lines through surface A is greater than the density of lines through surface B. Therefore, the electric field is more intense on surface A than on surface B. Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.

Representative electric field lines for the field due to a single positive point charge are shown in Figure 23.20a. Note that in this two-dimensional drawing we show only the field lines that lie in the plane containing the point charge. The lines are actually directed radially outward from the charge in all directions; thus, instead of the flat "wheel" of lines shown, you should picture an entire sphere of lines. Because a positive test charge placed in this field would be repelled by the positive point charge, the lines are directed radially away from the positive point

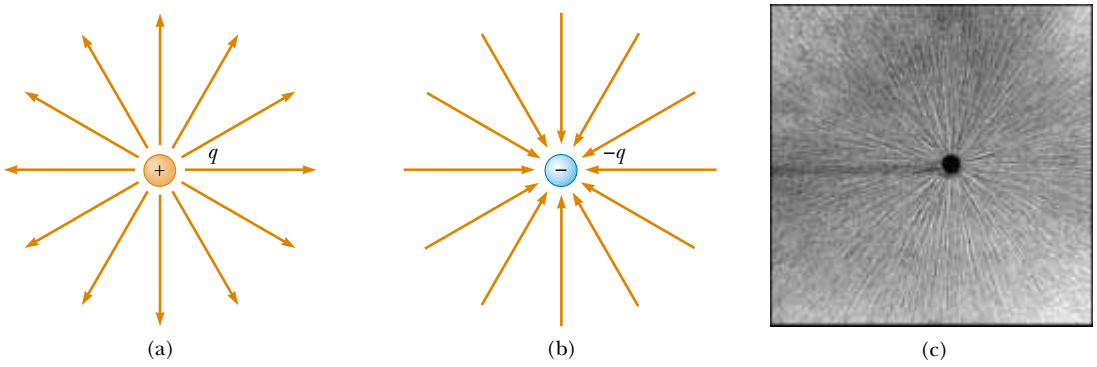


Figure 23.20 The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane containing the charge. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.

charge. The electric field lines representing the field due to a single negative point charge are directed toward the charge (Fig. 23.20b). In either case, the lines are along the radial direction and extend all the way to infinity. Note that the lines become closer together as they approach the charge; this indicates that the strength of the field increases as we move toward the source charge.

The rules for drawing electric field lines are as follows:

Rules for drawing electric field lines

- The lines must begin on a positive charge and terminate on a negative charge.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

Is this visualization of the electric field in terms of field lines consistent with Equation 23.4, the expression we obtained for E using Coulomb's law? To answer this question, consider an imaginary spherical surface of radius r concentric with a point charge. From symmetry, we see that the magnitude of the electric field is the same everywhere on the surface of the sphere. The number of lines N that emerge from the charge is equal to the number that penetrate the spherical surface. Hence, the number of lines per unit area on the sphere is $N/4\pi r^2$ (where the surface area of the sphere is $4\pi r^2$). Because E is proportional to the number of lines per unit area, we see that E varies as $1/r^2$; this finding is consistent with Equation 23.4.

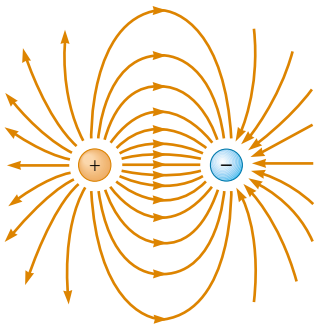
As we have seen, we use electric field lines to qualitatively describe the electric field. One problem with this model is that we always draw a finite number of lines from (or to) each charge. Thus, it appears as if the field acts only in certain directions; this is not true. Instead, the field is *continuous*—that is, it exists at every point. Another problem associated with this model is the danger of gaining the wrong impression from a two-dimensional drawing of field lines being used to describe a three-dimensional situation. Be aware of these shortcomings every time you either draw or look at a diagram showing electric field lines.

We choose the number of field lines starting from any positively charged object to be $C'q$ and the number of lines ending on any negatively charged object to be $C'|q|$, where C' is an arbitrary proportionality constant. Once C' is chosen, the number of lines is fixed. For example, if object 1 has charge Q_1 and object 2 has charge Q_2, then the ratio of number of lines is $N_2/N_1 = Q_2/Q_1$.

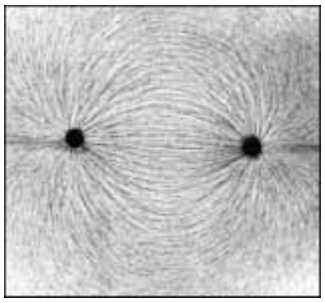
The electric field lines for two point charges of equal magnitude but opposite signs (an electric dipole) are shown in Figure 23.21. Because the charges are of equal magnitude, the number of lines that begin at the positive charge must equal the number that terminate at the negative charge. At points very near the charges, the lines are nearly radial. The high density of lines between the charges indicates a region of strong electric field.

Figure 23.22 shows the electric field lines in the vicinity of two equal positive point charges. Again, the lines are nearly radial at points close to either charge, and the same number of lines emerge from each charge because the charges are equal in magnitude. At great distances from the charges, the field is approximately equal to that of a single point charge of magnitude $2q$.

Finally, in Figure 23.23 we sketch the electric field lines associated with a positive charge $+2q$ and a negative charge $-q$. In this case, the number of lines leaving $+2q$ is twice the number terminating at $-q$. Hence, only half of the lines that leave the positive charge reach the negative charge. The remaining half terminate

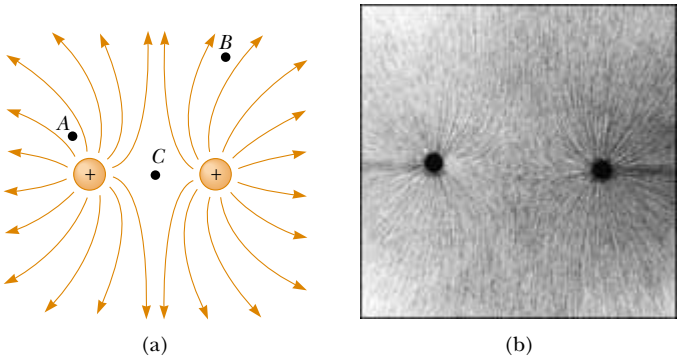


(a)



(b)

Figure 23.21 (a) The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole). The number of lines leaving the positive charge equals the number terminating at the negative charge. (b) The dark lines are small pieces of thread suspended in oil, which align with the electric field of a dipole.



(a) (b)

Figure 23.22 (a) The electric field lines for two positive point charges. (The locations *A*, *B*, and *C* are discussed in Quick Quiz 23.5.) (b) Pieces of thread suspended in oil, which align with the electric field created by two equal-magnitude positive charges.

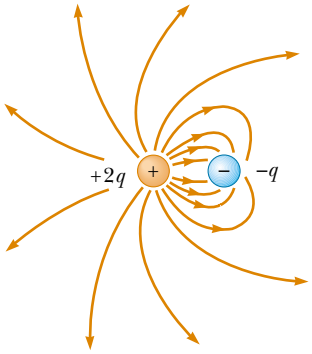


Figure 23.23 The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave $+2q$ for every one that terminates on $-q$.

on a negative charge we assume to be at infinity. At distances that are much greater than the charge separation, the electric field lines are equivalent to those of a single charge $+q$.

Quick Quiz 23.5

Rank the magnitude of the electric field at points *A*, *B*, and *C* shown in Figure 23.22a (greatest magnitude first).

23.7 MOTION OF CHARGED PARTICLES IN A UNIFORM ELECTRIC FIELD

When a particle of charge q and mass m is placed in an electric field $\mathbf{E}$, the electric force exerted on the charge is $q\mathbf{E}$. If this is the only force exerted on the particle, it must be the net force and so must cause the particle to accelerate. In this case, Newton's second law applied to the particle gives

$$\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$$

The acceleration of the particle is therefore

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \tag{23.7}$$

If $\mathbf{E}$ is uniform (that is, constant in magnitude and direction), then the acceleration is constant. If the particle has a positive charge, then its acceleration is in the direction of the electric field. If the particle has a negative charge, then its acceleration is in the direction opposite the electric field.

Example 23.10 An Accelerating Positive Charge

A positive point charge q of mass m is released from rest in a uniform electric field $\mathbf{E}$ directed along the x axis, as shown in Figure 23.24. Describe its motion.

Solution The acceleration is constant and is given by $q\mathbf{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one

dimension (see Chapter 2):

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$v_{xf} = v_{xi} + a_x t$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Taking $x_i = 0$ and $v_{xi} = 0$, we have

$$x_f = \tfrac{1}{2}a_x t^2 = \frac{qE}{2m}t^2$$

$$v_{xf} = a_x t = \frac{qE}{m}t$$

$$v_{xf}^2 = 2a_x x_f = \left(\frac{2qE}{m}\right)x_f$$

The kinetic energy of the charge after it has moved a distance $x = x_f - x_i$ is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)x = qEx$$

We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_e x = qEx$ and $W = \Delta K$.

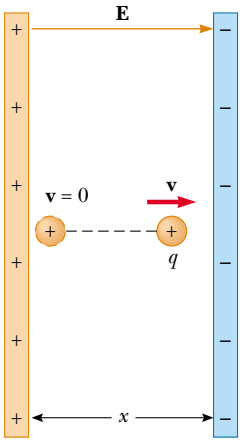


Figure 23.24 A positive point charge q in a uniform electric field **E** undergoes constant acceleration in the direction of the field.

The electric field in the region between two oppositely charged flat metallic plates is approximately uniform (Fig. 23.25). Suppose an electron of charge $-e$ is projected horizontally into this field with an initial velocity $v_i\mathbf{i}$. Because the electric field **E** in Figure 23.25 is in the positive y direction, the acceleration of the electron is in the negative y direction. That is,

$$\mathbf{a} = -\frac{eE}{m}\mathbf{j} \qquad (23.8)$$

Because the acceleration is constant, we can apply the equations of kinematics in two dimensions (see Chapter 4) with $v_{xi} = v_i$ and $v_{yi} = 0$. After the electron has been in the electric field for a time t, the components of its velocity are

$$v_x = v_i = \text{constant} \qquad (23.9)$$

$$v_y = a_y t = -\frac{eE}{m}t \qquad (23.10)$$

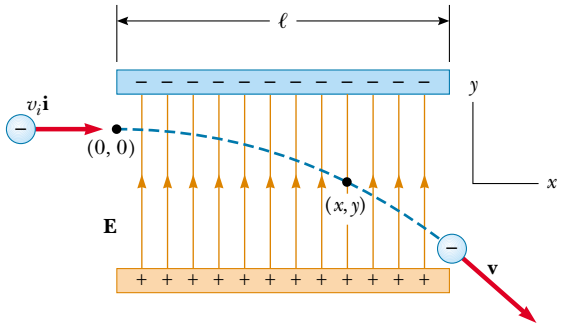


Figure 23.25 An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite **E**), and its motion is parabolic while it is between the plates.

Its coordinates after a time t in the field are

$$x = v_i t \tag{23.11}$$

$$y = \tfrac{1}{2} a_y t^2 = -\tfrac{1}{2} \frac{eE}{m} t^2 \tag{23.12}$$

Substituting the value $t = x/v_i$ from Equation 23.11 into Equation 23.12, we see that y is proportional to x^2. Hence, the trajectory is a parabola. After the electron leaves the field, it continues to move in a straight line in the direction of **v** in Figure 23.25, obeying Newton's first law, with a speed $v > v_i$.

Note that we have neglected the gravitational force acting on the electron. This is a good approximation when we are dealing with atomic particles. For an electric field of 10^4 N/C, the ratio of the magnitude of the electric force eE to the magnitude of the gravitational force mg is of the order of 10^{14} for an electron and of the order of 10^{11} for a proton.

EXAMPLE 23.11 An Accelerated Electron

An electron enters the region of a uniform electric field as shown in Figure 23.25, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m. (a) Find the acceleration of the electron while it is in the electric field.

Solution The charge on the electron has an absolute value of 1.60×10^{-19} C, and $m = 9.11 \times 10^{-31}$ kg. Therefore, Equation 23.8 gives

$$\mathbf{a} = -\frac{eE}{m}\mathbf{j} = -\frac{(1.60 \times 10^{-19}\ \text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\mathbf{j}$$

$$= \boxed{-3.51 \times 10^{13}\mathbf{j}\ \text{m/s}^2}$$

(b) Find the time it takes the electron to travel through the field.

Solution The horizontal distance across the field is $\ell = 0.100$ m. Using Equation 23.11 with $x = \ell$, we find that the time spent in the electric field is

$$t = \frac{\ell}{v_i} = \frac{0.100\ \text{m}}{3.00 \times 10^6\ \text{m/s}} = \boxed{3.33 \times 10^{-8}\ \text{s}}$$

(c) What is the vertical displacement y of the electron while it is in the field?

Solution Using Equation 23.12 and the results from parts (a) and (b), we find that

$$y = \tfrac{1}{2} a_y t^2 = \tfrac{1}{2}(-3.51 \times 10^{13}\ \text{m/s}^2)(3.33 \times 10^{-8}\ \text{s})^2$$

$$= -0.019\,5\ \text{m} = \boxed{-1.95\ \text{cm}}$$

If the separation between the plates is less than this, the electron will strike the positive plate.

Exercise Find the speed of the electron as it emerges from the field.

Answer 3.22×10^6 m/s.

The Cathode Ray Tube

The example we just worked describes a portion of a cathode ray tube (CRT). This tube, illustrated in Figure 23.26, is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems, television receivers, and computer monitors. The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields. The electron beam is produced by an assembly called an *electron gun* located in the neck of the tube. These electrons, if left undisturbed, travel in a straight-line path until they strike the front of the CRT, the "screen," which is coated with a material that emits visible light when bombarded with electrons.

In an oscilloscope, the electrons are deflected in various directions by two sets of plates placed at right angles to each other in the neck of the tube. (A television

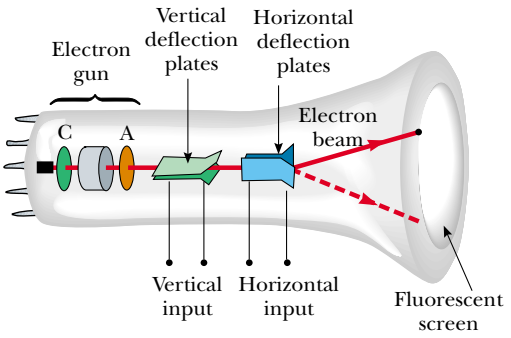


Figure 23.26 Schematic diagram of a cathode ray tube. Electrons leaving the hot cathode C are accelerated to the anode A. In addition to accelerating electrons, the electron gun is also used to focus the beam of electrons, and the plates deflect the beam.

CRT steers the beam with a magnetic field, as discussed in Chapter 29.) An external electric circuit is used to control the amount of charge present on the plates. The placing of positive charge on one horizontal plate and negative charge on the other creates an electric field between the plates and allows the beam to be steered from side to side. The vertical deflection plates act in the same way, except that changing the charge on them deflects the beam vertically.

SUMMARY

Electric charges have the following important properties:

- Unlike charges attract one another, and like charges repel one another.
- Charge is conserved.
- Charge is quantized—that is, it exists in discrete packets that are some integral multiple of the electronic charge.

Conductors are materials in which charges move freely. **Insulators** are materials in which charges do not move freely.

Coulomb's law states that the electric force exerted by a charge q_1 on a second charge q_2 is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} \tag{23.2}$$

where r is the distance between the two charges and $\hat{\mathbf{r}}$ is a unit vector directed from q_1 to q_2. The constant k_e, called the Coulomb constant, has the value $k_e = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

The smallest unit of charge known to exist in nature is the charge on an electron or proton, $|e| = 1.602\ 19 \times 10^{-19}\ \mathrm{C}$.

The electric field $\mathbf{E}$ at some point in space is defined as the electric force $\mathbf{F}_e$ that acts on a small positive test charge placed at that point divided by the magnitude of the test charge q_0:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \tag{23.3}$$

At a distance r from a point charge q, the electric field due to the charge is given by

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \tag{23.4}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the charge to the point in question. The

electric field is directed radially outward from a positive charge and radially inward toward a negative charge.

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \tag{23.5}$$

The electric field at some point of a continuous charge distribution is

$$\mathbf{E} = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \tag{23.6}$$

where dq is the charge on one element of the charge distribution and r is the distance from the element to the point in question.

Electric field lines describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of $\mathbf{E}$ in that region.

A charged particle of mass m and charge q moving in an electric field $\mathbf{E}$ has an acceleration

$$\mathbf{a} = \frac{q\mathbf{E}}{m} \tag{23.7}$$

Problem-Solving Hints

Finding the Electric Field

- **Units:** In calculations using the Coulomb constant k_e $(= 1/4\pi\epsilon_0)$, charges must be expressed in coulombs and distances in meters.
- **Calculating the electric field of point charges:** To find the total electric field at a given point, first calculate the electric field at the point due to each individual charge. The resultant field at the point is the vector sum of the fields due to the individual charges.
- **Continuous charge distributions:** When you are confronted with problems that involve a continuous distribution of charge, the vector sums for evaluating the total electric field at some point must be replaced by vector integrals. Divide the charge distribution into infinitesimal pieces, and calculate the vector sum by integrating over the entire charge distribution. You should review Examples 23.7 through 23.9.
- **Symmetry:** With both distributions of point charges and continuous charge distributions, take advantage of any symmetry in the system to simplify your calculations.

Questions

1. Sparks are often observed (or heard) on a dry day when clothes are removed in the dark. Explain.
2. Explain from an atomic viewpoint why charge is usually transferred by electrons.
3. A balloon is negatively charged by rubbing and then clings to a wall. Does this mean that the wall is positively charged? Why does the balloon eventually fall?
4. A light, uncharged metallic sphere suspended from a thread is attracted to a charged rubber rod. After touching the rod, the sphere is repelled by the rod. Explain.

5. Explain what is meant by the term "a neutral atom."
6. Why do some clothes cling together and to your body after they are removed from a dryer?
7. A large metallic sphere insulated from ground is charged with an electrostatic generator while a person standing on an insulating stool holds the sphere. Why is it safe to do this? Why wouldn't it be safe for another person to touch the sphere after it has been charged?
8. What are the similarities and differences between Newton's law of gravitation, $F_g = Gm_1m_2/r^2$, and Coulomb's law, $F_e = k_e q_1 q_2/r^2$?
9. Assume that someone proposes a theory that states that people are bound to the Earth by electric forces rather than by gravity. How could you prove this theory wrong?
10. How would you experimentally distinguish an electric field from a gravitational field?
11. Would life be different if the electron were positively charged and the proton were negatively charged? Does the choice of signs have any bearing on physical and chemical interactions? Explain.
12. When defining the electric field, why is it necessary to specify that the magnitude of the test charge be very small (that is, why is it necessary to take the limit of $\mathbf{F}_e/q$ as $q \rightarrow 0$)?
13. Two charged conducting spheres, each of radius a, are separated by a distance $r > 2a$. Is the force on either sphere given by Coulomb's law? Explain. (*Hint:* Refer to Chapter 14 on gravitation.)
14. When is it valid to approximate a charge distribution by a point charge?
15. Is it possible for an electric field to exist in empty space? Explain.
16. Explain why electric field lines never cross. (*Hint:* $\mathbf{E}$ must have a unique direction at all points.)
17. A free electron and free proton are placed in an identical electric field. Compare the electric forces on each particle. Compare their accelerations.
18. Explain what happens to the magnitude of the electric field of a point charge as r approaches zero.
19. A negative charge is placed in a region of space where the electric field is directed vertically upward. What is the direction of the electric force experienced by this charge?
20. A charge $4q$ is a distance r from a charge $-q$. Compare the number of electric field lines leaving the charge $4q$ with the number entering the charge $-q$.
21. In Figure 23.23, where do the extra lines leaving the charge $+2q$ end?
22. Consider two equal point charges separated by some distance d. At what point (other than ∞) would a third test charge experience no net force?
23. A negative point charge $-q$ is placed at the point P near the positively charged ring shown in Figure 23.17. If $x \ll a$, describe the motion of the point charge if it is released from rest.
24. Explain the differences between linear, surface, and volume charge densities, and give examples of when each would be used.
25. If the electron in Figure 23.25 is projected into the electric field with an arbitrary velocity $\mathbf{v}_i$ (at an angle to $\mathbf{E}$), will its trajectory still be parabolic? Explain.
26. It has been reported that in some instances people near where a lightning bolt strikes the Earth have had their clothes thrown off. Explain why this might happen.
27. Why should a ground wire be connected to the metallic support rod for a television antenna?
28. A light strip of aluminum foil is draped over a wooden rod. When a rod carrying a positive charge is brought close to the foil, the two parts of the foil stand apart. Why? What kind of charge is on the foil?
29. Why is it more difficult to charge an object by rubbing on a humid day than on a dry day?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*
WEB = solution posted at **http://www.saunderscollege.com/physics/** = Computer useful in solving problem = Interactive Physics
▭ = paired numerical/symbolic problems

Section 23.1 Properties of Electric Charges
Section 23.2 Insulators and Conductors
Section 23.3 Coulomb's Law

1. (a) Calculate the number of electrons in a small, electrically neutral silver pin that has a mass of 10.0 g. Silver has 47 electrons per atom, and its molar mass is 107.87 g/mol. (b) Electrons are added to the pin until the net negative charge is 1.00 mC. How many electrons are added for every 10^9 electrons already present?
2. (a) Two protons in a molecule are separated by a distance of 3.80×10^{-10} m. Find the electric force exerted by one proton on the other. (b) How does the magnitude of this force compare with the magnitude of the gravitational force between the two protons? (c) What must be the charge-to-mass ratio of a particle if the magnitude of the gravitational force between two of these particles equals the magnitude of the electric force between them?

WEB 3. Richard Feynman once said that if two persons stood at arm's length from each other and each person had 1% more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order-of-magnitude calculation to substantiate this assertion.

4. Two small silver spheres, each with a mass of 10.0 g, are separated by 1.00 m. Calculate the fraction of the elec-

trons in one sphere that must be transferred to the other to produce an attractive force of 1.00×10^4 N (about 1 ton) between the spheres. (The number of electrons per atom of silver is 47, and the number of atoms per gram is Avogadro's number divided by the molar mass of silver, 107.87 g/mol.)

5. Suppose that 1.00 g of hydrogen is separated into electrons and protons. Suppose also that the protons are placed at the Earth's north pole and the electrons are placed at the south pole. What is the resulting compressional force on the Earth?

6. Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC, and the other is given a charge of − 18.0 nC. (a) Find the electric force exerted on one sphere by the other. (b) The spheres are connected by a conducting wire. Find the electric force between the two after equilibrium has occurred.

7. Three point charges are located at the corners of an equilateral triangle, as shown in Figure P23.7. Calculate the net electric force on the 7.00-μC charge.

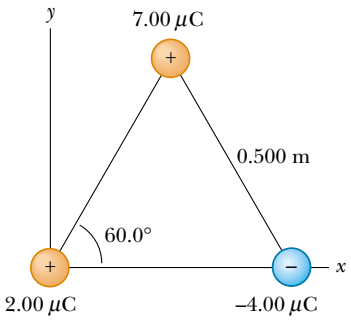


Figure P23.7 Problems 7 and 15.

8. Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal insulating rod extending from the origin to the point $x = d$. As shown in Figure P23.8, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

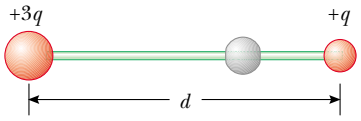


Figure P23.8

9. **Review Problem.** In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. (a) Find the electric force between the two. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

10. **Review Problem.** Two identical point charges each having charge $+q$ are fixed in space and separated by a distance d. A third point charge $-Q$ of mass m is free to move and lies initially at rest on a perpendicular bisector of the two fixed charges a distance x from the midpoint of the two fixed charges (Fig. P23.10). (a) Show that if x is small compared with d, the motion of $-Q$ is simple harmonic along the perpendicular bisector. Determine the period of that motion. (b) How fast will the charge $-Q$ be moving when it is at the midpoint between the two fixed charges, if initially it is released at a distance $x = a \ll d$ from the midpoint?

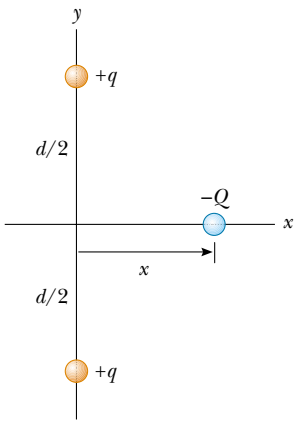


Figure P23.10

Section 23.4 The Electric Field

11. What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton? (Use the data in Table 23.1.)

12. An object having a net charge of 24.0 μC is placed in a uniform electric field of 610 N/C that is directed vertically. What is the mass of this object if it "floats" in the field?

13. In Figure P23.13, determine the point (other than infinity) at which the electric field is zero.

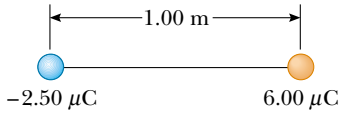


Figure P23.13

14. An airplane is flying through a thundercloud at a height of 2 000 m. (This is a very dangerous thing to do because of updrafts, turbulence, and the possibility of electric discharge.) If there are charge concentrations of + 40.0 C at a height of 3 000 m within the cloud and of − 40.0 C at a height of 1 000 m, what is the electric field E at the aircraft?

15. Three charges are at the corners of an equilateral triangle, as shown in Figure P23.7. (a) Calculate the electric field at the position of the 2.00-μC charge due to the 7.00-μC and $-$ 4.00-μC charges. (b) Use your answer to part (a) to determine the force on the 2.00-μC charge.

16. Three point charges are arranged as shown in Figure P23.16. (a) Find the vector electric field that the 6.00-nC and $-$ 3.00-nC charges together create at the origin. (b) Find the vector force on the 5.00-nC charge.

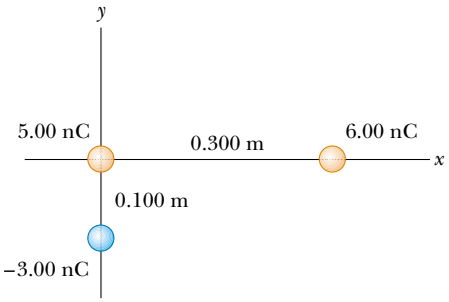


Figure P23.16

WEB **17.** Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (*Hint:* Sketch the field lines in the plane of the charges.) (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

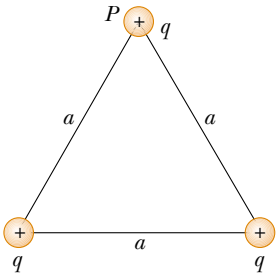


Figure P23.17

18. Two 2.00-μC point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric field on the y axis at $y = 0.500$ m. (b) Calculate the electric force on a $-$ 3.00-μC charge placed on the y axis at $y = 0.500$ m.

19. Four point charges are at the corners of a square of side a, as shown in Figure P23.19. (a) Determine the magnitude and direction of the electric field at the location of charge q. (b) What is the resultant force on q?

20. A point particle having charge q is located at point (x_0, y_0) in the xy plane. Show that the x and y components of the electric field at point (x, y) due to this charge q are

$$E_x = \frac{k_e q(x - x_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}$$

$$E_y = \frac{k_e q(y - y_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}$$

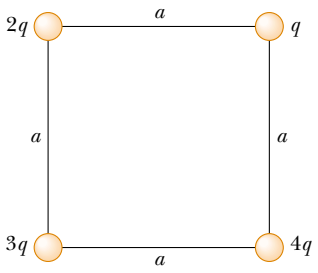


Figure P23.19

21. Consider the electric dipole shown in Figure P23.21. Show that the electric field at a *distant* point along the x axis is $E_x \cong 4k_e qa/x^3$.

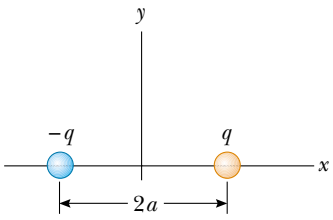


Figure P23.21

22. Consider n equal positive point charges each of magnitude Q/n placed symmetrically around a circle of radius R. (a) Calculate the magnitude of the electric field E at a point a distance x on the line passing through the center of the circle and perpendicular to the plane of the circle. (b) Explain why this result is identical to the one obtained in Example 23.8.

23. Consider an infinite number of identical charges (each of charge q) placed along the x axis at distances a, $2a$, $3a$, $4a$, . . . from the origin. What is the electric field at the origin due to this distribution? *Hint:* Use the fact that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$$

Section 23.5 Electric Field of a Continuous Charge Distribution

24. A rod 14.0 cm long is uniformly charged and has a total charge of $-$ 22.0 μC. Determine the magnitude and direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

25. A continuous line of charge lies along the x axis, extending from $x = +x_0$ to positive infinity. The line carries a uniform linear charge density λ_0. What are the magnitude and direction of the electric field at the origin?

26. A line of charge starts at $x = +x_0$ and extends to positive infinity. If the linear charge density is $\lambda = \lambda_0 x_0 / x$, determine the electric field at the origin.

27. A uniformly charged ring of radius 10.0 cm has a total charge of 75.0 μC. Find the electric field on the axis of the ring at (a) 1.00 cm, (b) 5.00 cm, (c) 30.0 cm, and (d) 100 cm from the center of the ring.

28. Show that the maximum field strength E_{max} along the axis of a uniformly charged ring occurs at $x = a/\sqrt{2}$ (see Fig. 23.17) and has the value $Q/(6\sqrt{3}\pi\epsilon_0 a^2)$.

29. A uniformly charged disk of radius 35.0 cm carries a charge density of 7.90×10^{-3} C/m^2. Calculate the electric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the center of the disk.

30. Example 23.9 derives the exact expression for the electric field at a point on the axis of a uniformly charged disk. Consider a disk of radius $R = 3.00$ cm having a uniformly distributed charge of $+5.20$ μC. (a) Using the result of Example 23.9, compute the electric field at a point on the axis and 3.00 mm from the center. Compare this answer with the field computed from the near-field approximation $E = \sigma/2\epsilon_0$. (b) Using the result of Example 23.9, compute the electric field at a point on the axis and 30.0 cm from the center of the disk. Compare this result with the electric field obtained by treating the disk as a $+5.20$-μC point charge at a distance of 30.0 cm.

31. The electric field along the axis of a uniformly charged disk of radius R and total charge Q was calculated in Example 23.9. Show that the electric field at distances x that are great compared with R approaches that of a point charge $Q = \sigma\pi R^2$. (*Hint:* First show that $x/(x^2 + R^2)^{1/2} = (1 + R^2/x^2)^{-1/2}$, and use the binomial expansion $(1 + \delta)^n \approx 1 + n\delta$ when $\delta \ll 1$.)

32. A piece of Styrofoam having a mass m carries a net charge of $-q$ and floats above the center of a very large horizontal sheet of plastic that has a uniform charge density on its surface. What is the charge per unit area on the plastic sheet?

WEB **33.** A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as shown in Figure P23.33. The rod has a total charge of -7.50 μC. Find the magnitude and direction of the electric field at O, the center of the semicircle.

34. (a) Consider a uniformly charged right circular cylindrical shell having total charge Q, radius R, and height h. Determine the electric field at a point a distance d from the right side of the cylinder, as shown in Figure P23.34. (*Hint:* Use the result of Example 23.8 and treat the cylinder as a collection of ring charges.) (b) Consider now a solid cylinder with the same dimensions and carrying the same charge, which is uniformly distributed through its volume. Use the result of Example 23.9 to find the field it creates at the same point.

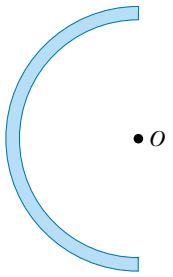


Figure P23.33

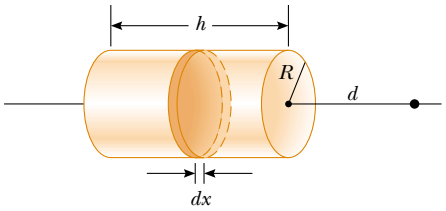


Figure P23.34

35. A thin rod of length ℓ and uniform charge per unit length λ lies along the x axis, as shown in Figure P23.35. (a) Show that the electric field at P, a distance y from the rod, along the perpendicular bisector has no x component and is given by $E = 2k_e\lambda \sin\theta_0/y$. (b) Using your result to part (a), show that the field of a rod of infinite length is $E = 2k_e\lambda/y$. (*Hint:* First calculate the field at P due to an element of length dx, which has a charge $\lambda\, dx$. Then change variables from x to θ, using the facts that $x = y \tan\theta$ and $dx = y \sec^2\theta\, d\theta$, and integrate over θ.)

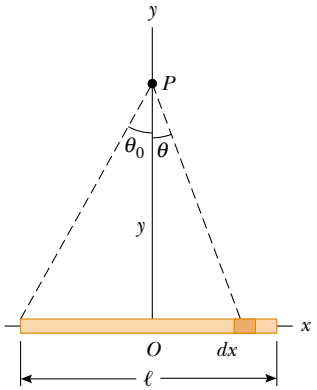


Figure P23.35

36. Three solid plastic cylinders all have a radius of 2.50 cm and a length of 6.00 cm. One (a) carries charge with

uniform density 15.0 nC/m^2 everywhere on its surface. Another (b) carries charge with the same uniform density on its curved lateral surface only. The third (c) carries charge with uniform density 500 nC/m^3 throughout the plastic. Find the charge of each cylinder.

37. Eight solid plastic cubes, each 3.00 cm on each edge, are glued together to form each one of the objects (i, ii, iii, and iv) shown in Figure P23.37. (a) If each object carries charge with a uniform density of 400 nC/m^3 throughout its volume, what is the charge of each object? (b) If each object is given charge with a uniform density of 15.0 nC/m^2 everywhere on its exposed surface, what is the charge on each object? (c) If charge is placed only on the edges where perpendicular surfaces meet, with a uniform density of 80.0 pC/m, what is the charge of each object?

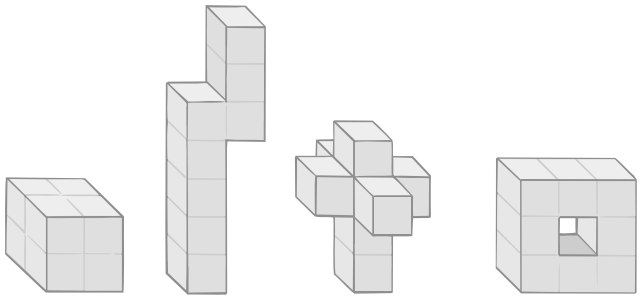


Figure P23.37

Section 23.6 Electric Field Lines

38. A positively charged disk has a uniform charge per unit area as described in Example 23.9. Sketch the electric field lines in a plane perpendicular to the plane of the disk passing through its center.

39. A negatively charged rod of finite length has a uniform charge per unit length. Sketch the electric field lines in a plane containing the rod.

40. Figure P23.40 shows the electric field lines for two point charges separated by a small distance. (a) Determine the ratio q_1/q_2. (b) What are the signs of q_1 and q_2?

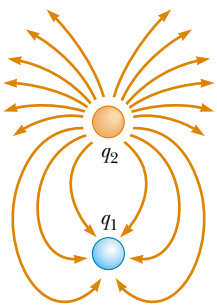


Figure P23.40

Section 23.7 Motion of Charged Particles in a Uniform Electric Field

41. An electron and a proton are each placed at rest in an electric field of 520 N/C. Calculate the speed of each particle 48.0 ns after being released.

42. A proton is projected in the positive x direction into a region of uniform electric field $\mathbf{E} = -6.00 \times 10^5 \mathbf{i}$ N/C. The proton travels 7.00 cm before coming to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time it takes the proton to come to rest.

43. A proton accelerates from rest in a uniform electric field of 640 N/C. At some later time, its speed has reached 1.20×10^6 m/s (nonrelativistic, since v is much less than the speed of light). (a) Find the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it moved in this time? (d) What is its kinetic energy at this time?

44. The electrons in a particle beam each have a kinetic energy of 1.60×10^{-17} J. What are the magnitude and direction of the electric field that stops these electrons in a distance of 10.0 cm?

WEB **45.** The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that stops these electrons in a distance d?

46. A positively charged bead having a mass of 1.00 g falls from rest in a vacuum from a height of 5.00 m in a uniform vertical electric field with a magnitude of 1.00×10^4 N/C. The bead hits the ground at a speed of 21.0 m/s. Determine (a) the direction of the electric field (up or down) and (b) the charge on the bead.

47. A proton moves at 4.50×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find (a) the time it takes the proton to travel 5.00 cm horizontally, (b) its vertical displacement after it has traveled 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

48. An electron is projected at an angle of 30.0° above the horizontal at a speed of 8.20×10^5 m/s in a region where the electric field is $\mathbf{E} = 390\mathbf{j}$ N/C. Neglecting the effects of gravity, find (a) the time it takes the electron to return to its initial height, (b) the maximum height it reaches, and (c) its horizontal displacement when it reaches its maximum height.

49. Protons are projected with an initial speed $v_i = 9.55 \times 10^3$ m/s into a region where a uniform electric field $\mathbf{E} = (-720\mathbf{j})$ N/C is present, as shown in Figure P23.49. The protons are to hit a target that lies at a horizontal distance of 1.27 mm from the point where the protons are launched. Find (a) the two projection angles θ that result in a hit and (b) the total time of flight for each trajectory.

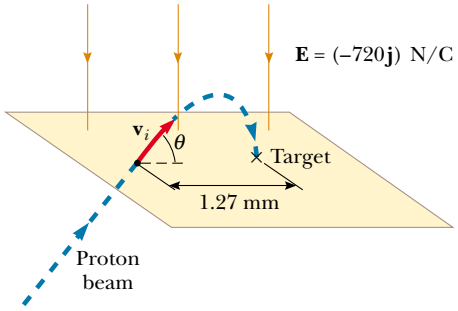


Figure P23.49

ADDITIONAL PROBLEMS

50. Three point charges are aligned along the x axis as shown in Figure P23.50. Find the electric field at (a) the position (2.00, 0) and (b) the position (0, 2.00).

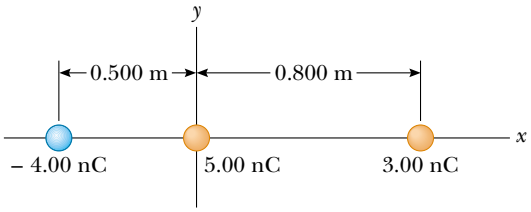


Figure P23.50

51. A uniform electric field of magnitude 640 N/C exists between two parallel plates that are 4.00 cm apart. A proton is released from the positive plate at the same instant that an electron is released from the negative plate. (a) Determine the distance from the positive plate at which the two pass each other. (Ignore the electrical attraction between the proton and electron.) (b) Repeat part (a) for a sodium ion (Na^+) and a chlorine ion (Cl^-).

52. A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field, as shown in Figure P23.52. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

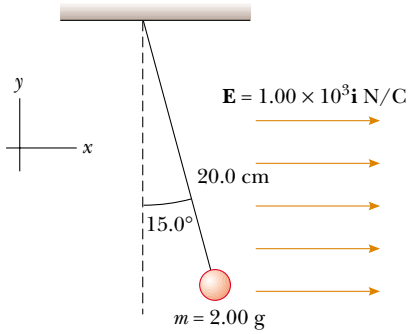


Figure P23.52

WEB **53.** A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N/C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

54. A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (A\mathbf{i} + B\mathbf{j})$ N/C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find (a) the charge on the ball and (b) the tension in the string.

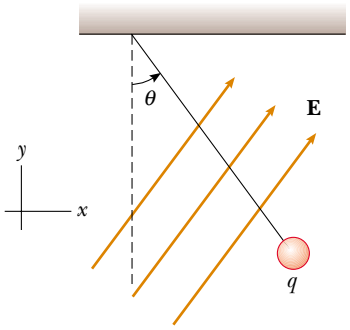


Figure P23.53 Problems 53 and 54.

55. Four identical point charges ($q = +10.0\ \mu$C) are located on the corners of a rectangle, as shown in Figure P23.55. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the magnitude and direction of the net electric force exerted on the charge at the lower left corner by the other three charges.

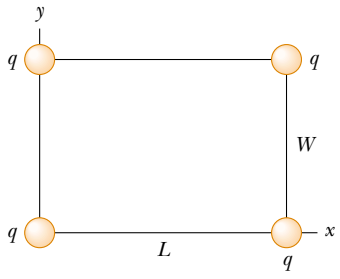


Figure P23.55

56. Three identical small Styrofoam balls ($m = 2.00$ g) are suspended from a fixed point by three nonconducting threads, each with a length of 50.0 cm and with negligi-

ble mass. At equilibrium the three balls form an equilateral triangle with sides of 30.0 cm. What is the common charge q carried by each ball?

57. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having the spring constant $k = 100$ N/m and an unstretched length of 0.300 m, as shown in Figure P23.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length of 0.400 m, as shown in Figure P23.57b. Determine the value of Q, assuming that all the charge resides on the blocks and that the blocks are like point charges.

58. Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having a spring constant k and an unstretched length L_i, as shown in Figure P23.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length L, as shown in Figure P23.57b. Determine the value of Q, assuming that all the charge resides on the blocks and that the blocks are like point charges.

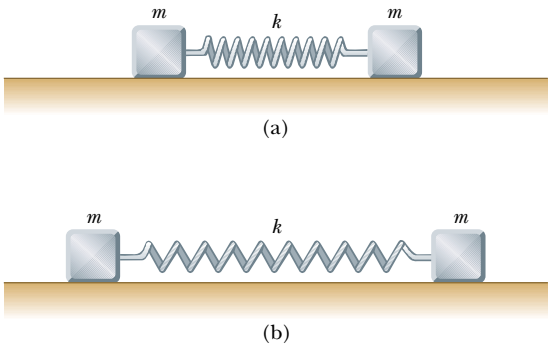


Figure P23.57 Problems 57 and 58.

59. Identical thin rods of length $2a$ carry equal charges, $+Q$, uniformly distributed along their lengths. The rods lie along the x axis with their centers separated by a distance of $b > 2a$ (Fig. P23.59). Show that the magnitude of the force exerted by the left rod on the right one is given by

$$F = \left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2 - 4a^2}\right)$$

60. A particle is said to be nonrelativistic as long as its speed is less than one-tenth the speed of light, or less than 3.00×10^7 m/s. (a) How long will an electron remain nonrelativistic if it starts from rest in a region of an electric field of 1.00 N/C? (b) How long will a proton remain nonrelativistic in the same electric field? (c) Electric fields are commonly much larger than

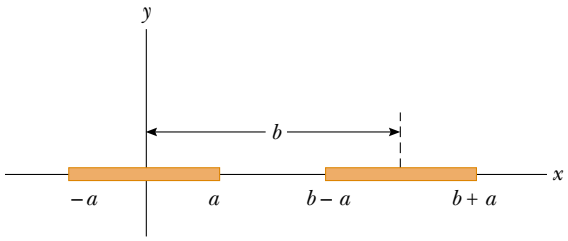


Figure P23.59

1 N/C. Will the charged particle remain nonrelativistic for a shorter or a longer time in a much larger electric field?

61. A line of positive charge is formed into a semicircle of radius $R = 60.0$ cm, as shown in Figure P23.61. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos\theta$. The total charge on the semicircle is 12.0 μC. Calculate the total force on a charge of 3.00 μC placed at the center of curvature.

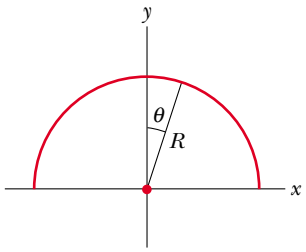


Figure P23.61

62. Two small spheres, each of mass 2.00 g, are suspended by light strings 10.0 cm in length (Fig. P23.62). A uniform electric field is applied in the x direction. The spheres have charges equal to -5.00×10^{-8} C and $+5.00 \times 10^{-8}$ C. Determine the electric field that enables the spheres to be in equilibrium at an angle of $\theta = 10.0°$.

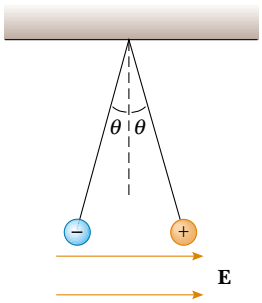


Figure P23.62

63. Two small spheres of mass m are suspended from strings of length ℓ that are connected at a common point. One sphere has charge Q; the other has charge $2Q$. Assume that the angles θ_1 and θ_2 that the strings make with the vertical are small. (a) How are θ_1 and θ_2 related? (b) Show that the distance r between the spheres is

$$r \cong \left(\frac{4k_e Q^2 \ell}{mg} \right)^{1/3}$$

64. Three charges of equal magnitude q are fixed in position at the vertices of an equilateral triangle (Fig. P23.64). A fourth charge Q is free to move along the positive x axis under the influence of the forces exerted by the three fixed charges. Find a value for s for which Q is in equilibrium. You will need to solve a transcendental equation.

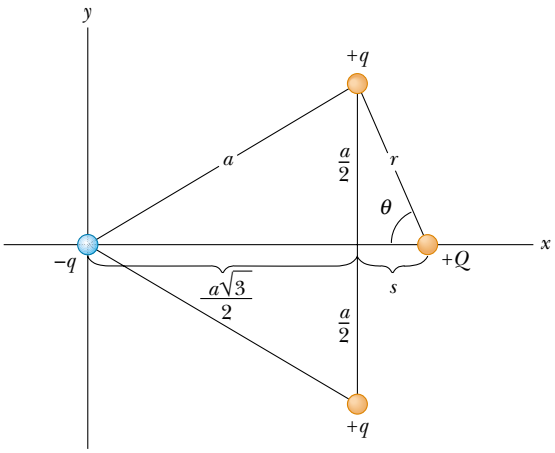


Figure P23.64

65. Review Problem. Four identical point charges, each having charge $+q$, are fixed at the corners of a square of side L. A fifth point charge $-Q$ lies a distance z along the line perpendicular to the plane of the square and passing through the center of the square (Fig. P23.65). (a) Show that the force exerted on $-Q$ by the other four charges is

$$\mathbf{F} = -\frac{4k_e qQz}{\left(z^2 + \frac{L^2}{2}\right)^{3/2}}\,\mathbf{k}$$

Note that this force is directed toward the center of the square whether z is positive ($-Q$ above the square) or negative ($-Q$ below the square). (b) If z is small compared with L, the above expression reduces to $\mathbf{F} \approx -(\text{constant})\,z\mathbf{k}$. Why does this imply that the motion of $-Q$ is simple harmonic, and what would be the period of this motion if the mass of $-Q$ were m?

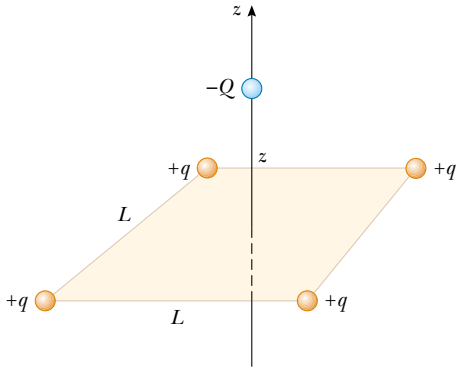


Figure P23.65

66. Review Problem. A 1.00-g cork ball with a charge of 2.00 μC is suspended vertically on a 0.500-m-long light string in the presence of a uniform, downward-directed electric field of magnitude $E = 1.00 \times 10^5$ N/C. If the ball is displaced slightly from the vertical, it oscillates like a simple pendulum. (a) Determine the period of this oscillation. (b) Should gravity be included in the calculation for part (a)? Explain.

67. Three charges of equal magnitude q reside at the corners of an equilateral triangle of side length a (Fig. P23.67). (a) Find the magnitude and direction of the electric field at point P, midway between the negative charges, in terms of k_e, q, and a. (b) Where must a $-4q$ charge be placed so that any charge located at P experiences no net electric force? In part (b), let P be the origin and let the distance between the $+q$ charge and P be 1.00 m.

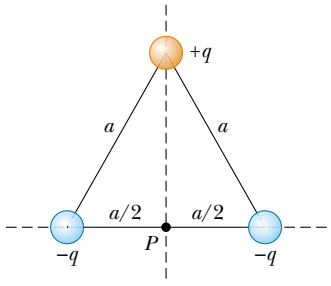


Figure P23.67

68. Two identical beads each have a mass m and charge q. When placed in a hemispherical bowl of radius R with frictionless, nonconducting walls, the beads move, and at equilibrium they are a distance R apart (Fig. P23.68). Determine the charge on each bead.

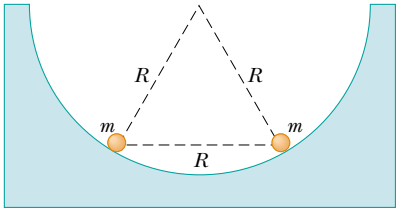


Figure P23.68

69. Eight point charges, each of magnitude q, are located on the corners of a cube of side s, as shown in Figure P23.69. (a) Determine the x, y, and z components of the resultant force exerted on the charge located at point A by the other charges. (b) What are the magnitude and direction of this resultant force?

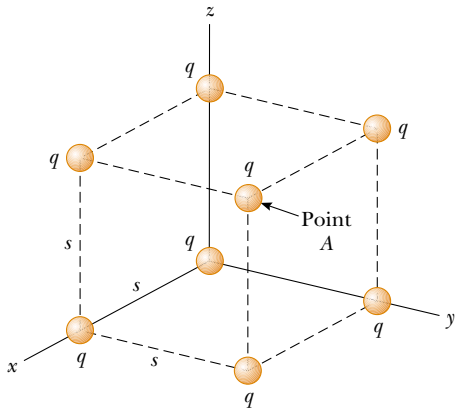


Figure P23.69 Problems 69 and 70.

70. Consider the charge distribution shown in Figure P23.69. (a) Show that the magnitude of the electric field at the center of any face of the cube has a value of $2.18k_e q/s^2$. (b) What is the direction of the electric field at the center of the top face of the cube?

71. A line of charge with a uniform density of 35.0 nC/m lies along the line $y = -15.0$ cm, between the points with coordinates $x = 0$ and $x = 40.0$ cm. Find the electric field it creates at the origin.

72. Three point charges q, $-2q$, and q are located along the x axis, as shown in Figure P23.72. Show that the electric field at P ($y \gg a$) along the y axis is

$$\mathbf{E} = -k_e \frac{3qa^2}{y^4}\mathbf{j}$$

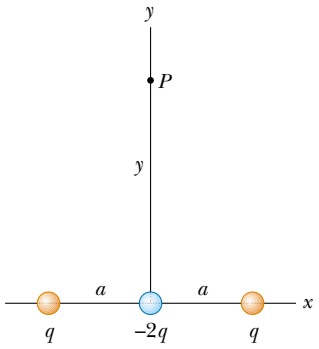


Figure P23.72

This charge distribution, which is essentially that of two electric dipoles, is called an *electric quadrupole*. Note that $\mathbf{E}$ varies as r^{-4} for the quadrupole, compared with variations of r^{-3} for the dipole and r^{-2} for the monopole (a single charge).

73. **Review Problem.** A negatively charged particle $-q$ is placed at the center of a uniformly charged ring, where the ring has a total positive charge Q, as shown in Example 23.8. The particle, confined to move along the x axis, is displaced a *small* distance x along the axis (where $x \ll a$) and released. Show that the particle oscillates with simple harmonic motion with a frequency

$$f = \frac{1}{2\pi}\left(\frac{k_e qQ}{ma^3}\right)^{1/2}$$

74. **Review Problem.** An electric dipole in a uniform electric field is displaced slightly from its equilibrium position, as shown in Figure P23.74, where θ is small and the charges are separated by a distance $2a$. The moment of inertia of the dipole is I. If the dipole is released from this position, show that its angular orientation exhibits simple harmonic motion with a frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{2qaE}{I}}$$

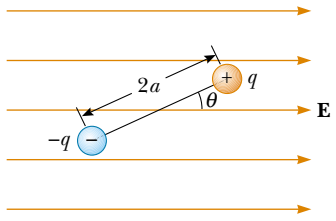
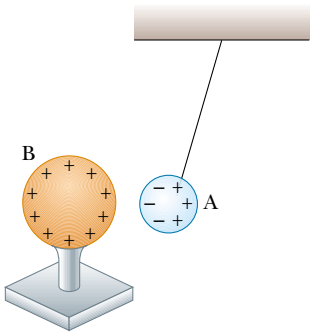


Figure P23.74

Answers to Quick Quizzes

23.1 (b). The amount of charge present after rubbing is the same as that before; it is just distributed differently.

23.2 (d). Object A might be negatively charged, but it also might be electrically neutral with an induced charge separation, as shown in the following figure:



23.3 (b). From Newton's third law, the electric force exerted by object B on object A is equal in magnitude to the force exerted by object A on object B and in the opposite direction—that is, $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$.

23.4 Nothing, if we assume that the source charge producing the field is not disturbed by our actions. Remember that the electric field is created not by the $+3\text{-}\mu\text{C}$ charge or by the $-3\text{-}\mu\text{C}$ charge but by the source charge (unseen in this case).

23.5 *A*, *B*, and *C*. The field is greatest at point *A* because this is where the field lines are closest together. The absence of lines at point *C* indicates that the electric field there is zero.